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Roo, M. de; Holten, J.W. van; Wit, B. de; Proeyen, A. Van

Published in:
Nuclear Physics B

DOI:
10.1016/0550-3213(80)90449-6

IMPORTANT NOTE: You are advised to consult the publisher's version (publisher's PDF) if you wish to cite from it. Please check the document version below.

Document Version
Publisher's PDF, also known as Version of record

Publication date:
1980

Link to publication in University of Groningen/UMCG research database

Citation for published version (APA):
Roo, M. D., Holten, J. W. V., Wit, B. D., & Proeyen, A. V. (1980). Chiral superfields in N = 2 supergravity. *Nuclear Physics B*, *173*(1). https://doi.org/10.1016/0550-3213(80)90449-6

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Download date: 05-05-2023

Nuclear Physics B173 (1980) 175–188
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CHIRAL SUPERFIELDS IN $N=2$ SUPERGRAVITY

M. DE ROO and J.W. VAN HOLTEN
Instituut Lorentz, University of Leiden, Nieuwsteeg 18, Leiden, The Netherlands

B. DE WIT
NIKHEF-H, P.O. Box 41882, 1009 DB Amsterdam, The Netherlands

A. VAN PROEYEN[1]
Instituut voor Theoretische Fysica, K.U. Leuven, Celestijnenlaan 200D, B-3030 Leuven, Belgium

Received 17 March 1980

The transformation laws of chiral (scalar) superfields with arbitrary Weyl weight w are determined for the U(2) superconformal theory. A superconformally invariant density is given for fields with $w=2$. For $w=1$ it is possible to have smaller irreducible multiplets. The full restriction upon which the chiral superfield becomes reducible is exhibited. These results define a complete calculus for the construction of invariant actions with chiral superfields. As an example we find the action for the vector gauge multiplet.

1. Introduction

In a previous paper [1] we have given the full transformation rules for $N=2$ Poincaré supergravity in the formulation with auxiliary fields [2, 3]. At the same time we have obtained transformations for the various submultiplets, contained in the Poincaré fields, namely, the Weyl multiplet, the vector gauge multiplet and the tensor gauge multiplet. The Weyl multiplet contains the highest-spin components, and is the field representation of the U(2) extended conformal theory. To understand the relation between this multiplet and the gauge fields of the superconformal algebra requires knowledge of the constraints; a set of such constraints was indeed presented in [1]. Use of the superconformal notions then proved helpful to clarify the transformation rules of the lower-spin submultiplets. In particular, the uniform decomposition of Poincaré supersymmetry into field-dependent superconformal transformations allows a separate treatment of each of the superconformal transformations, thereby leading to a more systematic understanding of the Poincaré transformations. Furthermore, the full algebra of superconformal transformations is implicit in our results.

This motivated us to study superconformal transformations for a larger variety of multiplets. The purpose of this paper is to give the full superconformal

[1] Aspirant NFWO.

transformations on chiral (scalar) $N=2$ superfields. Such fields, which have $16+16$ (fermionic + bosonic) components, occur with an arbitrary Weyl and chiral weight denoted by w. Products of equal-chirality superfields are again chiral superfields, also when the transformations to all orders in the gravitational coupling constant are included. (We keep referring to this constant κ to enable a direct comparison to the Poincaré theory.) The chiral superfield is a representation of the superconformal algebra, and generates a superconformal "tensor calculus". If $w=2$ it is possible to define an invariant density. We also discuss the case of reduced chiral superfields, which have only $8+8$ components. Such superfields must necessarily have $w=1$. Their relevance stems from the fact that all three submultiplets of the Poincaré field representation can be interpreted within the context of reduced chiral multiplets.

A superconformal tensor calculus is an intermediate step towards a full tensor calculus for SO(2) extended Poincaré supergravity. This has already been demonstrated for $N=1$ [4]. The crucial element to enable the extension to Poincaré supergravity is the previously mentioned decomposition rule for Poincaré supersymmetry in terms of superconformal symmetries. Since the Weyl weight will then lose its relevance one may redefine the fields to obtain the proper Poincaré components, whose transformations no longer refer to a specific weight. This will be discussed in a separate publication [5]. Results for Poincaré supermultiplets can also be found in ref. [6].

This paper is organized as follows. In sect. 2 we discuss chiral scalar superfields for rigid $N=2$ supersymmetry, and review some relevant aspects of conformal supergravity. In sect. 3 we extend the chiral superfield to a full representation of the superconformal algebra. We also give the formula for an invariant density, and discuss reduced chiral superfields. As an example we present the invariant action of the vector gauge multiplet [7, 8], which describes the particle content $(1, \frac{1}{2}, \frac{1}{2}, 0^+, 0^-)$, in sect. 4. We give our conclusions in sect. 5.

2. Preliminaries

2.1. CHIRAL SUPERFIELDS

Superfields [9] are elements of a Grassmann algebra on the basis of N anticommuting Majorana spinors θ, labelled by indices $i=1,2,\dots N$. We will adopt a general chiral SU(N) notation throughout this paper which employs chiral and self-dual components [10]. Namely, the left-handed chiral component of θ is assigned to the fundamental representation of SU(N), and carries an upper SU(N) index; right-handed components will then transform according to the conjugate representation, in agreement with the Majorana property and have a lower index. Hence we have

$$\begin{aligned}(1-\gamma_5)\theta^i&=(1+\gamma_5)\theta_i=0\,,\\ \bar{\theta}^i(1-\gamma_5)&=\bar{\theta}_i(1+\gamma_5)=0\,.\end{aligned}\tag{2.1}$$

We will use this notation consistently, so that chirality and the SU(N) transformation properties are in direct correspondence, even although for $N=2$, the fundamental representation and its conjugate are equivalent.

A left-handed chiral superfield $\Phi^{(+)}$ is defined by the condition that the right-handed operation

$$D^i \equiv \frac{\partial}{\partial \bar{\theta}_i} + \gamma^\mu \theta^i \frac{\partial}{\partial x^\mu} \tag{2.2}$$

vanishes when applied to $\Phi^{(+)}$. This implies that the dependence of $\Phi^{(+)}$ on θ_i is contained only through a complex spacetime parameter z^μ, defined by

$$z^\mu = x^\mu + \bar{\theta}^i \gamma^\mu \theta_i \,. \tag{2.3}$$

Therefore $\Phi^{(+)}$ can be decomposed on the basis of the left-handed generators θ^i only, in terms of 2^{2N} complex functions of z^μ. For $N=2$ we thus find the following decomposition:

$$\begin{aligned} \Phi^{(+)}(z^\mu, \theta^i) = A(z) + \bar{\theta}^i \Psi_i(z) + \tfrac{1}{2}\bar{\theta}^i \theta^j B_{ij}(z) + \tfrac{1}{2}(\epsilon_{ij}\bar{\theta}^i \sigma_{ab}\theta^j) F^-_{ab}(z) \\ + \tfrac{1}{3}(\epsilon_{ij}\bar{\theta}^i \sigma_{ab}\theta^j)\bar{\theta}^k \sigma^{ab} \Lambda_k(z) + \tfrac{1}{12}(\epsilon_{ij}\bar{\theta}^i \sigma_{ab}\theta^j)^2 C(z) \,. \end{aligned} \tag{2.4}$$

The bosonic components A, B_{ij}, F^-_{ab} and C are complex, and the fermionic components Ψ_i and Λ_i are left-handed chiral (Majorana) spinors. The field F^-_{ab} is anti-self-dual: $\tilde{F}^- = -F^-$. The complex conjugate of $\Phi^{(+)}$ is precisely a right-handed chiral superfield, satisfying

$$D_i \Phi^{(-)} = 0 \,, \tag{2.5}$$

with

$$D_i \equiv \frac{\partial}{\partial \bar{\theta}^i} + \gamma^\mu \theta_i \frac{\partial}{\partial x^\mu} \,.$$

Therefore, $\Phi^{(-)}$ can be decomposed on the basis of right-handed generators θ_i, in terms of functions of $(z^\mu)^*$. As is well-known, chiral superfields can be re-expressed in terms of functions of x^μ by means of

$$\begin{aligned} \Phi^{(+)}(z^\mu, \theta^i) &= \exp(-\bar{\theta}_i \,\partial\!\!\!/\, \theta^i) \Phi^{(+)}(x^\mu, \theta^i) \,, \\ \Phi^{(-)}((z^\mu)^*, \theta_i) &= \exp(-\bar{\theta}^i \,\partial\!\!\!/\, \theta_i) \Phi^{(-)}(x^\mu, \theta_i) \,. \end{aligned} \tag{2.6}$$

In principle one has the possibility of considering chiral superfields with extra external SU(N) or local Lorentz indices. But in this paper we only deal with the singlet version. Such fields have been discussed in the past [11, 12], and here we only summarize a number of aspects that are relevant for subsequent sections.

Within the superconformal context it is meaningful to assign Weyl and chiral weights to a chiral superfield. It follows from the full superconformal algebra, that these weights are related. They determine the transformation properties under dilatations and chiral U(1) transformations. More precisely, a chiral superfield with weight w transforms according to

$$\Phi^{(+)}(z^{\mu}, \theta^{i}) \to e^{w\zeta^{*}} \Phi^{(+)}(z^{\mu}, e^{-\zeta/2}\theta^{i}), \tag{2.7}$$

where ζ is a complex number

$$\zeta = \zeta_{W} + i\zeta_{A},$$

with ζ_W and ζ_A the transformation parameters of the dilatations and chiral transformations, respectively.

It is obvious from the previous discussion that the product of two left (right)-handed chiral superfields with weights w_1 and w_2, is again a left (right)-handed superfield with weight $w_3 = w_1 + w_2$. We give the expression for the product of two $N=2$ superfields in terms of components. If $(A, \Psi, B, F, \Lambda, C)$ denote the components of a left-handed chiral superfield then

$$\begin{aligned}
&(A, \Psi_i, B_{ij}, F^{-}_{ab}, \Lambda_i, C) \otimes (a, \psi_i, b_{ij}, f^{-}_{ab}, \lambda_i, c) \\
&\quad = (Aa,\ a\Psi_i + A\psi_i,\ aB_{ij} + Ab_{ij} - \tfrac{1}{2}\bar{\Psi}_{(i}\psi_{j)}, \\
&\qquad aF^{-}_{ab} + Af^{-}_{ab} - \tfrac{1}{2}\epsilon^{ij}\bar{\Psi}_i\sigma_{ab}\psi_j, \\
&\qquad a\Lambda_i + A\lambda_i - \tfrac{1}{2}\epsilon^{kl}(B_{ik}\psi_l + b_{ik}\Psi_l) - \tfrac{1}{2}(\sigma\cdot f^{-}\Psi_i + \sigma\cdot F^{-}\psi_i), \\
&\qquad aC + Ac - \tfrac{1}{2}\epsilon^{ik}\epsilon^{jl}B_{ij}b_{kl} + F^{-}_{ab}f^{-}_{ab} + \epsilon^{ij}(\bar{\Psi}_i\lambda_j + \bar{\psi}_i\Lambda_j)).
\end{aligned} \tag{2.8}$$

The rigid supersymmetry transformation of a chiral superfield can also be determined straightforwardly in terms of components

$$\begin{aligned}
\delta A &= \bar{\epsilon}^{i}\Psi_i, \\
\delta \Psi_i &= 2\,\not{\partial} A\epsilon_i + B_{ij}\epsilon^{j} + \sigma\cdot F^{-}\epsilon^{j}\epsilon_{ij}, \\
\delta B_{ij} &= \bar{\epsilon}_{(i}\,\not{\partial}\Psi_{j)} - \bar{\epsilon}^{k}\Lambda_{(i}\epsilon_{j)k}, \\
\delta F^{-}_{ab} &= \epsilon^{ij}\bar{\epsilon}_i\,\not{\partial}\sigma_{ab}\Psi_j + \bar{\epsilon}^{i}\sigma_{ab}\Lambda_i, \\
\delta \Lambda_i &= -\sigma\cdot F^{-}\,\overleftarrow{\not{\partial}}\epsilon_i - \not{\partial} B_{ij}\epsilon_k\epsilon^{jk} + C\epsilon^{j}\epsilon_{ij}, \\
\delta C &= -2\epsilon^{ij}\bar{\epsilon}_i\,\not{\partial}\Lambda_j.
\end{aligned} \tag{2.9}$$

Notice that we have also used a chiral notation for the transformation parameters ϵ.

An important aspect of chiral superfields in $N=2$ is that for a certain weight it is possible to reduce the number of independent components. This is achieved by imposing the following SU(2) covariant restriction:

$$(\epsilon_{ij}\bar{D}^{i}\sigma_{ab}D^{j})^{2}(\Phi^{(+)})^{*} = \mp 96\,\Box\,\Phi^{(+)}. \tag{2.10}$$

In terms of components, this constraint has the following consequences:

$$\begin{aligned} \Lambda_i &= \mp\epsilon_{ij}\,\partial\!\!\!/\Psi^j\,,\\ B_{ij} &= \pm\epsilon_{ik}\epsilon_{jl}B^{*kl}\,,\\ C &= \mp 2\,\Box A^*\,,\\ \partial_a F^-_{ab} &= \pm\partial_a F^+_{ab}\,. \end{aligned} \tag{2.11}$$

Notice that the last equation is a Bianchi identity, which implies that F [or $\tilde{F}$, depending on the sign in (2.10), (2.11)] can be expressed as a field strength in terms of a vector potential. By comparing weights of both sides of (2.10) it is obvious that this restriction can only be imposed for weight $w=1$. The vector gauge multiplet [7, 8] discussed in [1] is of this kind, and has indeed components with the corresponding weights. We will give the full correspondence later. The tensor gauge multiplet is also related to a reduced chiral superfield. For instance if we choose a minus sign in (2.10) the components of the tensor multiplet are related to the higher θ components of the superfield according to (compare ref. [1])

$$\begin{aligned} &\varphi^i = \epsilon_{ij}\Lambda^j + \epsilon^{ij}\Lambda_j\,,\\ &A\delta_{ij} + iB_{ij} = B_{ij}\,,\\ &E_{\mu\nu}{}^{ij} = i(F^+_{\mu\nu} - F^-_{\mu\nu})\epsilon^{ij}\,,\\ &F - iG = -C^*\,. \end{aligned} \tag{2.12}$$

Again the Weyl and chiral weights implied by eqs. (2.12) agree precisely with those found in [1].

As is already obvious from the assignment of Weyl and chiral weights to a chiral superfield, a product of two reduced superfields will be a chiral superfield of the general type, which will no longer satisfy the restriction (2.10). This observation completes the discussion of chiral superfields in flat superspace.

2.2. $N=2$ SUPERCONFORMAL GRAVITY

We now summarize some results of [1] in chiral notation. The superconformal theory contains the gauge fields e_μ^a, $\omega_\mu{}^{ab}$, b_μ, $f_\mu{}^a$, $\psi_\mu{}^i$, $\phi_\mu{}^i$, $\mathcal{A}_\mu$, $\mathcal{V}_\mu{}^i{}_j$, and "matter fields" $T^-_{ab}{}^{ij}$, χ_C^i, D_C. With upper SU(2) indices $\psi_\mu{}^i$ and $\chi_C{}^i$ denote left-handed chiral components, whereas $\phi_\mu{}^i$ is right-handed. In SU(2) notation, the SU(2) gauge fields are contained in an antihermitian quantity $\mathcal{V}_\mu{}^i{}_j$ defined in terms of the fields of [1] by

$$\mathcal{V}_\mu{}^i{}_j = (\mathcal{V}_\mu{}^{ij} - i\mathcal{A}_\mu{}^{ij})\,. \tag{2.13}$$

The gauge fields $\omega_\mu{}^{ab}$, $f_\mu{}^a$, and $\phi_\mu{}^i$, of local Lorentz, special conformal, and S-supersymmetry transformations are expressed in terms of the other fields by means of the constraints

$$\begin{aligned}
&R_{\mu\nu}{}^{a}(\mathrm{P})=0\,,\\
&\gamma^{a}(\hat{R}_{ab}{}^{i}(\mathrm{Q})+\sigma_{ab}\chi^{i}_{\mathrm{C}})=0\,,\\
&\hat{R}_{\mu\nu a}{}^{b}(\mathrm{M})e^{\nu}{}_{b}-i\kappa\hat{\tilde{R}}_{\mu a}(\mathscr{A})+\tfrac{1}{8}\kappa^{2}T^{+}_{abij}T^{-ij}_{\mu b}-\tfrac{3}{2}a\kappa D_{\mathrm{C}}e_{\mu a}=0\,.
\end{aligned}\tag{2.14}$$

The curvatures R_{ab} can be found in the tables of [1]. In the third constraint we have allowed a free parameter a.

Because of the constraints (2.14) the curvature tensors $\hat{R}(\mathrm{Q})$ are self-dual, whereas the modified curvature tensors $\mathscr{R}(\mathrm{S})$ are almost self-dual. The latter follows from the Bianchi identity (4.13) in ref. [1]. Hence, we have the following useful identities

$$\begin{aligned}
&\hat{R}_{ab}{}^{i}(\mathrm{Q})=-\hat{\tilde{R}}_{ab}{}^{i}(\mathrm{Q})=\gamma_{5}\hat{R}_{ab}{}^{i}(\mathrm{Q})\,,\\
&\mathscr{R}_{ab}{}^{i}(\mathrm{S})-\tilde{\mathscr{R}}_{ab}{}^{i}(\mathrm{S})=2\gamma^{c}\sigma_{ab}D^{\mathrm{C}}_{d}\hat{R}_{cd}{}^{i}(\mathrm{Q})\,,\\
&\sigma^{ab}\mathscr{R}_{ab}{}^{i}(\mathrm{S})=\tfrac{3}{2}\not{D}^{\mathrm{C}}\chi^{i}_{\mathrm{C}}\,.
\end{aligned}\tag{2.15}$$

We repeat the Q- and S-supersymmetry transformations of the superconformal fields in chiral notation

$$\begin{aligned}
\delta e_{\mu}{}^{a}&=\kappa\bar{\epsilon}^{i}\gamma^{a}\psi_{\mu i}+\text{h.c.}\,,\\
\delta\omega_{\mu}{}^{ab}&=-\kappa\bar{\epsilon}^{i}\sigma^{ab}\phi_{\mu i}+\tfrac{3}{2}\kappa\bar{\epsilon}^{i}\gamma_{\mu}\sigma^{ab}\chi_{\mathrm{C}i}+\kappa\bar{\epsilon}^{i}\gamma_{\mu}\hat{R}^{ab}{}_{i}(\mathrm{Q})\\
&\quad-\tfrac{1}{2}\kappa^{2}\bar{\epsilon}^{i}T^{+ab}{}_{ij}\psi_{\mu}{}^{j}+\kappa\bar{\psi}_{\mu}{}^{i}\sigma^{ab}\eta_{i}+\text{h.c.}\,,\\
\delta b_{\mu}&=-\tfrac{3}{4}\kappa\bar{\epsilon}^{i}\gamma_{\mu}\chi_{\mathrm{C}i}+\tfrac{1}{2}\kappa\bar{\epsilon}^{i}\phi_{\mu i}-\tfrac{1}{2}\kappa\bar{\psi}_{\mu}{}^{i}\eta_{i}+\text{h.c.}\,,\\
\delta f_{\mu}{}^{a}&=\kappa\bar{\epsilon}^{i}\gamma_{\mu}D^{\mathrm{C}}_{b}\hat{R}_{bai}(\mathrm{Q})-\tfrac{1}{4}\kappa e_{\mu}{}^{a}(a+2)\bar{\epsilon}^{i}\not{D}^{\mathrm{C}}\chi_{\mathrm{C}i}\\
&\quad-\tfrac{1}{2}\kappa^{2}\bar{\epsilon}^{i}\psi_{\mu}{}^{j}D^{\mathrm{C}}_{b}T^{+ba}{}_{ij}-\tfrac{1}{4}\kappa^{2}(a+2)\bar{\epsilon}^{i}\gamma^{a}\psi_{\mu i}D_{\mathrm{C}}+\tfrac{1}{2}\kappa\bar{\eta}^{i}\gamma^{a}\phi_{\mu i}+\text{h.c.}\,,\\
\delta\psi_{\mu}{}^{i}&=2\kappa^{-1}D_{\mu}{}^{\mathrm{W}}\epsilon^{i}-\tfrac{1}{4}\sigma\cdot T^{-ij}\gamma_{\mu}\epsilon_{j}-\kappa^{-1}\gamma_{\mu}\eta^{i}\,,\\
\delta\phi_{\mu}{}^{i}&=-2\kappa^{-1}f_{\mu}{}^{a}\gamma_{a}\epsilon^{i}+\tfrac{1}{2}\hat{R}(\mathscr{V})^{i}{}_{j}\cdot\sigma\gamma_{\mu}\epsilon^{j}+i\hat{R}(\mathscr{A})\cdot\sigma\gamma_{\mu}\epsilon^{i}\\
&\quad-\tfrac{1}{4}\not{D}^{\mathrm{C}}T^{-ij}\cdot\sigma\gamma_{\mu}\epsilon_{j}+\tfrac{1}{2}(1-a)D_{\mathrm{C}}\gamma_{\mu}\epsilon^{i}\\
&\quad+\tfrac{3}{2}\kappa((\bar{\chi}_{\mathrm{C}j}\gamma^{a}\epsilon^{j})\gamma_{a}\psi_{\mu}{}^{i}-(\bar{\chi}_{\mathrm{C}j}\gamma^{a}\psi_{\mu}{}^{j})\gamma_{a}\epsilon^{i})+2\kappa^{-1}D_{\mu}{}^{\mathrm{W}}\eta^{i}\,,\\
\delta\mathscr{A}_{\mu}&=\tfrac{3}{4}i\bar{\epsilon}^{i}\gamma_{\mu}\chi_{\mathrm{C}i}+\tfrac{1}{2}i\bar{\epsilon}^{i}\phi_{\mu i}-\tfrac{1}{2}i\bar{\psi}_{\mu}{}^{i}\eta_{i}+\text{h.c.}\,,\\
\delta\mathscr{V}_{\mu}{}^{i}{}_{j}&=3\bar{\epsilon}^{i}\gamma_{\mu}\chi_{\mathrm{C}j}-2\bar{\epsilon}^{i}\phi_{\mu j}+2\bar{\psi}_{\mu}{}^{i}\eta_{j}-(\text{h.c.; traceless})\,,\\
\delta T^{-ij}_{ab}&=4\bar{\epsilon}^{[i}\hat{R}_{ab}{}^{j]}(\mathrm{Q})\,,
\end{aligned}\tag{2.16}$$

$$\delta\chi^i_{\rm C} = -\tfrac{1}{6}\sigma\cdot T^{-ij}\hat{\not{D}}^{\rm C}\epsilon_j + \tfrac{1}{3}\hat{R}(\mathcal{V})^i{}_j\cdot\sigma\epsilon^j - \tfrac{2}{3}i\hat{R}(\mathcal{A})\cdot\sigma\epsilon^i + D_{\rm C}\epsilon^i + \tfrac{1}{6}\sigma\cdot T^{-ij}\eta_j\,,$$

$$\delta D_{\rm C} = \bar{\epsilon}^i\not{D}^{\rm C}\chi_{{\rm C}i} + \text{h.c.}\,, \tag{2.16}$$

where ϵ^i and η^i denote the parameters of Q- and S-supersymmetry, respectively. Notice that ϵ^i has left-handed and η^i right-handed chirality. We have used derivatives $D_\mu{}^{\rm C}$ that are fully covariantized with respect to the superconformal symmetries, except in $\delta\psi_\mu{}^i$ and $\delta\phi_\mu{}^i$, where $D_\mu{}^{\rm W}$ is defined by

$$D_\mu{}^{\rm W}\epsilon^i = (\partial_\mu - \tfrac{1}{2}\omega_\mu\cdot\sigma + \tfrac{1}{2}b_\mu + \tfrac{1}{2}i\kappa\mathcal{A}_\mu)\epsilon^i + \tfrac{1}{2}\kappa\mathcal{V}_\mu{}^i{}_j\epsilon^j\,,$$

$$D_\mu{}^{\rm W}\eta^i = (\partial_\mu - \tfrac{1}{2}\omega_\mu\cdot\sigma - \tfrac{1}{2}b_\mu + \tfrac{1}{2}i\kappa\mathcal{A}_\mu)\eta^i + \tfrac{1}{2}\kappa\mathcal{V}_\mu{}^i{}_j\eta^j\,. \tag{2.17}$$

On the basis of the transformation rules (2.16) one can verify that the superconformal transformations close under (anti)commutation. In particular, we find for the commutator of two supersymmetry transformations with parameter ϵ_1 and ϵ_2:

$$[\delta_{\rm Q}(\epsilon_1), \delta_{\rm Q}(\epsilon_2)] = 2(\bar{\epsilon}_2^k\gamma^a\epsilon_{1k} + \text{h.c.})D_a{}^{\rm C} + \delta_{\rm M}(\epsilon^{ab}) + \delta_{\rm K}(\Lambda^a_{\rm K}) + \delta_{\rm S}(\eta)\,, \tag{2.18a}$$

where $D_a{}^{\rm C}$ indicates the effect of a superconformally covariant translation [13], and ϵ^{ab}, $\Lambda^a_{\rm K}$ and η are the parameters of an extra field-dependent Lorentz, special conformal and S-supersymmetry transformation respectively

$$\epsilon^{ab} = \kappa(\bar{\epsilon}_1^i T^{+ab}{}_{ij}\epsilon_2{}^j + \text{h.c.})\,,$$

$$\Lambda^a_{\rm K} = \kappa\bar{\epsilon}_1^i D_b{}^{\rm C}T^+_{baij}\epsilon_2^j - \tfrac{1}{2}\kappa(a+2)D_{\rm C}\bar{\epsilon}_2^i\gamma_a\epsilon_{1i} + \text{h.c.}\,, \tag{2.18b}$$

$$\eta^i = 3\kappa\bar{\epsilon}_1^{[i}\epsilon_2^{j]}\chi_{{\rm C}j}\,.$$

The commutator of a Q- and S-supersymmetry transformation with parameters ϵ and η, respectively, is given by the superconformal algebra. Its precise form is

$$[\delta_{\rm S}(\eta), \delta_{\rm Q}(\epsilon)] = \delta_{\rm M}(\epsilon^{ab}) + \delta_{\rm D}(\zeta) + \delta_{\rm U(2)}(\Lambda^i{}_j)\,. \tag{2.19a}$$

The Lorentz, Weyl and U(2) chiral transformation on the r.h.s. are given by

$$\epsilon^{ab} = 2\bar{\eta}^j\sigma^{ab}\epsilon_j + \text{h.c.}\,, \qquad \zeta = \bar{\eta}^i\epsilon_i + \text{h.c.}\,,$$

$$\Lambda^i{}_j = -2\bar{\eta}^i\epsilon_j + \tfrac{1}{2}\delta^i{}_j\bar{\eta}^k\epsilon_k - \text{h.c.}\,, \tag{2.19b}$$

and the U(2) transformation is defined by its action on the field $\psi_\mu{}^i$:

$$\delta_{\rm U(2)}\psi_\mu{}^i = \Lambda^i{}_j\psi_\mu{}^j\,.$$

3. Chiral superfields in $N=2$ supergravity

The results of rigid supersymmetry for chiral superfields can be extended to the full supeconformal theory. This requires the definition of the supermultiplet as a representation of the superconformal algebra. In particular, the commutators (2.18) and (2.19) must be realized for all its components (2.4). The first step in obtaining this representation is to find the transformation rules under S-supersymmetry. This is most easily done by extending the derivatives in (2.9) to covariant derivatives with respect to dilatations, which generates extra terms in the transformation rules proportional to $\not{b}\epsilon_i$, depending on the Weyl weight of the multiplet. As we know from [1], these terms can be cancelled precisely by an S-supersymmetry transformation with parameter $\not{b}\epsilon_i$, which allows a direct identification of these transformations. They are

$$
\begin{aligned}
\delta_S A &= 0\,, \\
\delta_S \Psi_i &= 2wA\eta_i\,, \\
\delta_S B_{ij} &= (1-w)\bar{\eta}_{(i}\Psi_{j)}\,, \\
\delta_S F^-_{ab} &= -(1+w)\epsilon^{ij}\bar{\eta}_i\sigma_{ab}\Psi_j\,, \\
\delta_S \Lambda_i &= -(1+w)B_{ij}\epsilon^{jk}\eta_k + (1-w)\sigma\cdot F^-\eta_i\,, \\
\delta_S C &= 2w\epsilon^{ij}\bar{\eta}_i\Lambda_j\,.
\end{aligned}
\tag{3.1}
$$

These results agree with those of ref. [11]. Using (3.1) we can now fully covariantize the rigid supersymmetry transformations (2.9) with respect to the superconformal symmetries. One then realizes that these transformations are complete for the low-θ components, because no modifications can be found that are consistent with the assignments of Weyl and chiral weights. However, for the highest components Λ_i and C, modifications are possible, which contain the Weyl matter fields $T^+_{ab\,ij}$ and χ_{Ci}. Their precise form can be found by imposing the commutation relation (2.18) to all orders, making estensive use of (2.14)–(2.16). A straightforward calculation then leads to the full transformation rules under Q-supersymmetry

$$
\begin{aligned}
\delta_Q A &= \bar{\epsilon}^i\Psi_i\,, \\
\delta_Q \Psi_i &= 2\not{D}^C A\epsilon_i + B_{ij}\epsilon^j + \sigma\cdot F^-\epsilon_{ij}\epsilon^j\,, \\
\delta_Q B_{ij} &= \bar{\epsilon}_{(i}\not{D}^C\Psi_{j)} - \bar{\epsilon}^k\Lambda_{(i}\epsilon_{j)k}\,, \\
\delta_Q F^-_{ab} &= \epsilon^{ij}\bar{\epsilon}_i\not{D}^C\sigma_{ab}\Psi_j + \bar{\epsilon}^i\sigma_{ab}\Lambda_i\,, \\
\delta_Q \Lambda_i &= -\sigma\cdot F^-\overleftarrow{\not{D}}^C\epsilon_i - \not{D}^C B_{ij}\epsilon_k\epsilon^{jk} + C\epsilon^j\epsilon_{ij} \\
&\quad + \tfrac{1}{2}\kappa((\not{D}^C A)T^+_{ij}\cdot\sigma + wA\not{D}^C T^+_{ij}\cdot\sigma)\epsilon_k\epsilon^{jk} - \tfrac{3}{2}\kappa(\bar{\chi}_{C[i}\gamma_a\Psi_{j]})\gamma_a\epsilon_k\epsilon^{jk}\,, \\
\delta_Q C &= -2\epsilon^{ij}\bar{\epsilon}_i\not{D}^C\Lambda_j - 6\kappa\bar{\epsilon}_i\chi_{Cj}B_{kl}\epsilon^{ik}\epsilon^{jl} \\
&\quad - \tfrac{1}{2}\kappa\bar{\epsilon}_i((w-1)\sigma\cdot T^+_{jk}\overleftarrow{\not{D}}^C\Psi_l + \sigma\cdot T^+_{jk}\overrightarrow{\not{D}}^C\Psi_l)\epsilon^{ij}\epsilon^{kl}\,.
\end{aligned}
\tag{3.2}
$$

To verify the correctness of our results, we have also calculated the commutator of a Q- and an S-supersymmetry transformation and found complete agreement with (2.19). It also turns out that the non-covariant modifications in (3.2) are independent of the dilatational gauge field b_μ, so that our argument for finding (3.1) remains unaffected. Eq. (3.2) clarifies why non-covariant modifications were found in the tensor gauge multiplet, but not in the vector gauge multiplet [1], since the latter corresponds to the low-θ components of the chiral superfield. We will give a full discussion of the vector multiplet in sect. 4.

If $w=2$ the highest component C of the superfield has Weyl weight 4, and chiral weight 0. Therefore, C can serve as a starting point for the construction of an invariant action. Calculating order-by-order in κ leads to the following density which is invariant under all superconformal symmetries:

$$
\begin{aligned}
e^{-1}\mathscr{L} = C &- \kappa\epsilon^{ij}\bar{\psi}_i\cdot\gamma\Lambda_j - \tfrac{1}{4}\kappa^2\bar{\psi}_{\mu i}\sigma\cdot T^{+}_{jk}\gamma^{\mu}\Psi_l\epsilon^{ij}\epsilon^{kl} \\
&- \tfrac{1}{16}\kappa^2 A(T^{+}_{abij}\epsilon^{ij})^2 - \kappa^2\bar{\psi}_{\mu i}\sigma^{\mu\nu}\psi_{\nu j}B_{kl}\epsilon^{ik}\epsilon^{jl} \\
&+ \kappa^2\bar{\psi}_{\mu i}\psi_{\nu j}\epsilon^{ij}(F^{-\mu\nu} - \tfrac{1}{2}\kappa A T^{+\mu\nu}_{kl}\epsilon^{kl}) \\
&- \tfrac{1}{2}\kappa^3\epsilon^{ij}\epsilon^{kl}\, e^{-1}\epsilon^{\mu\nu\rho\sigma}\bar{\psi}_{\mu i}\psi_{\nu j}(\bar{\psi}_{\rho k}\gamma_{\sigma}\Psi_l + \kappa\bar{\psi}_{\rho k}\psi_{\sigma l}A)\,.
\end{aligned}
\tag{3.3}
$$

We have thus presented the main elements of a superconformal tensor calculus for chiral superfields. The multiplication rule for products of these superfields is still given by eq. (2.8). This is obvious for the low-θ components, because their transformation rules are superconformal covariantizations of the linearized results. For the full multiplet the multiplication property has been verified by direct computations.

These results also apply to the reduced chiral multiplet with $w=1$, but the reduction takes a more complicated form in a superconformal background. Namely, the constraints (2.11) are replaced by

$$
\begin{aligned}
\Lambda_i &= \mp\epsilon_{ij}\not{D}^{\mathrm{C}}\Psi^j\,, \\
B_{ij} &= \pm\epsilon_{ik}\epsilon_{jl}B^{*kl}\,, \\
C &= \mp(2\,\Box^{\mathrm{C}}A^* + \tfrac{1}{4}\kappa T^{+}_{abij}F^{+}_{ab}\epsilon^{ij} + 3\kappa\bar{\chi}_{\mathrm{C}i}\Psi^i - 2\kappa(1-a)D_{\mathrm{C}}A^*)\,, \\
D^{\mathrm{C}}_aF^{-}_{ab} &= \pm D^{\mathrm{C}}_aF^{+}_{ab} + \tfrac{1}{4}\kappa D^{\mathrm{C}}_a(\epsilon^{ij}T^{+}_{abij}A \mp \epsilon_{ij}T^{-ij}_{ab}A^*) \\
&\quad - \tfrac{3}{4}\kappa(\bar{\chi}_{\mathrm{C}i}\gamma_b\Psi_j\epsilon^{ij} \mp \bar{\chi}^i_{\mathrm{C}}\gamma_b\Psi^j\epsilon_{ij})\,.
\end{aligned}
\tag{3.4}
$$

The superconformal d'Alembertian is defined by

$$
\Box^{\mathrm{C}}A = D^{\mathrm{C}}_aD^{\mathrm{C}}_aA\,. \tag{3.5}
$$

Its form can immediately be obtained from the transformation properties of D^{C}_aA. In particular D^{C}_aA transforms under Q- and S-supersymmetry and special conformal boosts, with parameters ϵ^i, η^i and Λ^a_{K}, respectively, according to

$$
\begin{aligned}
\delta(D^{\mathrm{C}}_aA) = \bar{\epsilon}^iD^{\mathrm{C}}_a\Psi_i &- \tfrac{1}{8}\kappa\bar{\epsilon}_i\gamma_a\sigma\cdot T^{-ij}\Psi_j + \tfrac{3}{2}\kappa w\bar{\epsilon}_i\gamma_a\chi^i_{\mathrm{C}}A \\
&- \tfrac{1}{2}\bar{\eta}^i\gamma_a\Psi_i - w\Lambda_{\mathrm{K}a}A\,.
\end{aligned}
\tag{3.6}
$$

A direct way of obtaining (3.4) is to calculate the variation of $\not{D}^{C}\Psi_i$:

$$\delta_Q(\not{D}^{C}\Psi_i) = \not{D}^{C}\sigma \cdot F^{-}\epsilon^{j}\epsilon_{ij} + \not{D}^{C}B_{ij}\epsilon^{j}$$
$$+(2\,\Box^{C} A + \tfrac{1}{4}\kappa F^{-}_{ab}T^{-ij}_{ab}\epsilon_{ij} + 3\bar{\chi}^{i}_{C}\Psi_i - 2\kappa(1-a)D_{C}A)\epsilon_i$$
$$+\tfrac{1}{2}\kappa(AT^{+}_{ij}\cdot\sigma)\tilde{\not{D}}^{C}\epsilon^{j} + \tfrac{3}{2}\kappa(\bar{\chi}_{C[i}\gamma_a\Psi_{j]})\gamma^{a}\epsilon^{j}\,. \tag{3.7}$$

Assuming the first equation of (3.4), we compare the variation of $\mp\epsilon_{ij}\not{D}^{C}\Psi^{j}$ with $\delta_Q\Lambda_i$. This gives the remaining equations of (3.4). Subsequently one verifies the consistency of all four equations. Using (3.4) we can now discuss reduced chiral superfields within the same context as general chiral superfields. As an example of this we will construct the lagrangian of the vector gauge multiplet in the next section.

4. The vector gauge multiplet

The $N=2$ vector multiplet describes a massless vector gauge field V_μ, a complex scalar A, and a doublet of Majorana fermions Ψ_i. If the gauge field is represented through its modified field-strength tensor $F_{\mu\nu}$, then the components of this multiplet form a reduced chiral superfield [1]. More precisely, V_μ transforms according to

$$\delta V_\mu = \bar{\epsilon}_i\gamma_\mu\Psi_j\epsilon^{ij} + 2\kappa\bar{\epsilon}_i\psi_{\mu j}\epsilon^{ij}A + \text{h.c.}\,, \tag{4.1}$$

and has a modified field strength*

$$F_{\mu\nu} = \partial_\mu V_\nu - \partial_\nu V_\mu - \tfrac{1}{2}\kappa(\bar{\psi}_{[\mu i}\gamma_{\nu]}\Psi_j\epsilon^{ij} + \bar{\psi}^{i}_{[\mu}\gamma_{\nu]}\Psi^{j}\epsilon_{ij})$$
$$-\kappa^2(\bar{\psi}_{\mu i}\psi_{\nu j}\epsilon^{ij}A + \bar{\psi}_\mu{}^{i}\psi_\nu{}^{j}\epsilon_{ij}A^*) - \tfrac{1}{4}\kappa(AT^{+}_{\mu\nu ij}\epsilon^{ij} + A^*T^{-ij}_{\mu\nu}\epsilon_{ij})\,. \tag{4.2}$$

The components A, ψ_i, B_{ij}, $F^{-}_{ab} = \tfrac{1}{2}(F_{ab} - \tilde{F}_{ab})$ are the low-θ components of the superfield, whereas the high-θ components follow from (3.4). The invariant action of the vector multiplet can be found from the general chiral superfield $(\Phi^{(+)})^2$, where $\Phi^{(+)}$ represents the reduced chiral superfield of this multiplet. Since $(\Phi^{(+)})^2$ has weight $w=2$, one can immediately apply the density formula (3.3) to obtain the action. The components of $(\Phi^{(+)})^2$, denoted by $(a, \psi_i, b_{ij}, f^{-}_{ab}, \lambda_i, c)$ are equal to

$$a = A^2\,,$$
$$\psi_i = 2A\Psi_i\,,$$
$$b_{ij} = 2AB_{ij} - \bar{\Psi}_i\Psi_j\,, \tag{4.3}$$

* Our notation is adapted to the chiral-superfield decomposition. The precise relation to the components of [1] is given by

$$V_\mu = -W_\mu, \qquad B_{ij} = -\epsilon_{ik}(F - iG)^{k}{}_{j}\,,$$
$$\Psi_i = \tfrac{1}{2}(1+\gamma_5)\epsilon_{ij}\Psi^{j}, \qquad F_{ab} = -\mathcal{F}_{ab}(W)\,,$$
$$A = \tfrac{1}{4}\epsilon_{ij}(A - iB)^{ij}$$

$$
\begin{aligned}
f^-_{ab} &= 2AF^-_{ab} - \tfrac{1}{2}\epsilon^{ij}\bar{\Psi}_i\sigma_{ab}\Psi_j\,,\\
\lambda_i &= -2\epsilon_{ij}A\not{D}^{\mathrm{C}}\Psi^j - B_{ik}\Psi_l\epsilon^{kl} - \sigma\cdot F^-\Psi_i\,,\\
c &= -4A\,\Box^{\mathrm{C}}A^* + 4(1-a)\kappa D_{\mathrm{C}}|A|^2 - \tfrac{1}{2}\kappa AF^+_{ab}T^+_{abij}\epsilon^{ij}\\
&\quad -6\kappa A\bar{\chi}_{\mathrm{C}i}\Psi^i - \tfrac{1}{2}|B_{ij}|^2 + (F^-_{ab})^2 + 2\bar{\Psi}_i\not{D}^{\mathrm{C}}\Psi^i\,.
\end{aligned}
\tag{4.3}
$$

This can now be substituted into (3.3), which leads to the following invariant lagrangian (with proper normalization)

$$
\begin{aligned}
e^{-1}\mathcal{L} &= A\,\Box^{\mathrm{C}}A^* - \tfrac{1}{4}(F^-_{ab})^2 - \tfrac{1}{2}\bar{\Psi}_i\not{D}^{\mathrm{C}}\Psi^i + \tfrac{1}{8}|B_{ij}|^2\\
&\quad -\kappa(1-a)D_{\mathrm{C}}|A|^2 + \tfrac{1}{8}\kappa AF^+_{ab}T^+_{abij}\epsilon^{ij} + \tfrac{3}{2}\kappa A\bar{\chi}_{\mathrm{C}i}\Psi^i\\
&\quad +\tfrac{1}{2}\kappa\bar{\psi}_i\cdot\gamma\not{D}^{\mathrm{C}}\Psi^iA + \tfrac{1}{4}\kappa\bar{\psi}_i\cdot\gamma\Psi_jB^{*ij} - \tfrac{1}{4}\kappa\epsilon^{ij}\bar{\psi}_i\cdot\gamma\sigma\cdot F^-\Psi_j\\
&\quad +\tfrac{1}{64}\kappa^2A^2(T^+_{abij}\epsilon^{ij})^2 + \tfrac{1}{8}\kappa^2A\bar{\psi}_{\mu i}\sigma\cdot T^+_{jk}\gamma^\mu\Psi_l\epsilon^{ij}\epsilon^{kl}\\
&\quad +\tfrac{1}{4}\kappa^2\bar{\psi}_{\mu i}\sigma^{\mu\nu}\psi_{\nu j}(2AB^{*ij} - \epsilon^{ik}\epsilon^{jl}\bar{\Psi}_k\Psi_l)\\
&\quad -\tfrac{1}{4}\kappa^2\bar{\psi}_{ai}\psi_{bj}\epsilon^{ij}(2AF^-_{ab} - \tfrac{1}{2}\kappa A^2T^+_{abkl}\epsilon^{kl} - \tfrac{1}{2}\epsilon^{kl}\Psi_k\sigma_{ab}\Psi_l)\\
&\quad +\tfrac{1}{8}\kappa^3\epsilon^{ij}\epsilon^{kl}\,e^{-1}\epsilon^{\mu\nu\rho\sigma}\bar{\psi}_{\mu i}\psi_{\nu j}(2\bar{\psi}_{\rho k}\gamma_\sigma\Psi_lA + \kappa\bar{\psi}_{\rho k}\psi_{\sigma l}A^2)\\
&\quad +\text{h.c.}\,.
\end{aligned}
\tag{4.4}
$$

The lagrangian (4.4) can be coupled to $N=2$ Poincaré or Weyl supergravity. In both cases one can eliminate auxiliary fields, and obtain an invariant action in terms of dynamical fields. We will not do this in any detail, but rather confine ourselves to a qualitative discussion of the most salient features of (4.4).

We start by noticing that (4.4) seems to depend on the parameter a that was introduced in the constraint (2.14). This constraint expresses f^a_μ, the gauge field of the conformal boosts, in terms of other fields, and it receives an a-dependent contribution

$$
f^a_\mu = -\tfrac{1}{4}aD_{\mathrm{C}}\,e_\mu{}^a + \cdots\,. \tag{4.5}
$$

Because f^a_μ occurs in the definition of the superconformal d'Alembertian $\Box^{\mathrm{C}}$ [see (3.6)], this introduces a second a-dependent term in the action through (4.5). As expected, this precisely cancels the original term. A second point is that the auxiliary fields B_{ij} occur in various places in (4.4), both explicitly and through supercovariantizations. Apart from the term quadratic in B_{ij}, the dependence on B_{ij} disappears, which is in agreement with explicit Noether-type constructions of the lagrangian.

As mentioned before we can couple (4.4) to Weyl supergravity. This theory has only one auxiliary field, D_{C}, which occurs quadratically in the Weyl action [3]. Its elimination leads to a self-interaction

$$
e^{-1}\mathcal{L} = -\tfrac{1}{3}|A|^4\,. \tag{4.6}
$$

Of course, the lagrangian also contains the standard conformal improvement terms (supersymmetric generalizations of $R|A|^2$) for the scalar fields, which enter through the gauge field f_μ^a as well as through various supercovariantizations.

In the coupling to $N=2$ Poincaré supergravity many more auxiliary fields play a role, and their elimination leads to a large variety of non-polynomial modifications. The final results can be compared to the work of Luciani, who considered the coupling of several vector multiplets [14]. However, before the elimination can take place one must first represent the superconformal components in terms of the Poincaré fields. These substitutions are known and have been given in [1]. The most important one is for D_C, which in the Poincaré theory is equal to (not in chiral notation):

$$D_C = D^P \cdot V - \frac{1}{3\kappa} R^P + \tfrac{1}{4}\kappa((V_a^{ij})^2 + (A_a^{ij})^2 - 2V_a^2 - (M^{ij})^2 - (N^{ij})^2)$$
$$+ \kappa\bar{\lambda}^i(2\chi^i + 2\not{D}^P\lambda^i - \tfrac{1}{2}\gamma \cdot R^{iP} + i\kappa\gamma_5\not{A}\lambda^i - \kappa\sigma \cdot T^{ij}\lambda^j)\,. \tag{4.7}$$

The definition of the Poincaré components has been given in [1], and we remind the reader that the spinor χ^i, the vector V_a^{ij} and the axial vector A_a^{ij}, are related to the superconformal fields χ_C^i and $\mathcal{V}_{\mu j}^i$, respectively.

Because of the D_C-dependent term in (4.4), many auxiliary fields occur with a characteristic factor $(1-2\kappa^2|A|^2)$ in the coupling to Poincaré supergravity. To exhibit some of these terms, we give part of the combined action:

$$\mathcal{L}_{\text{Poincaré}} + \mathcal{L}_{\text{vector}} = -S^2 - \tfrac{1}{2}(P^{ij})^2 + \tfrac{1}{8}(t_{ab}^{ij})^2 + A_a^2$$
$$+ (1-2\kappa^2|A|^2)\{-\tfrac{1}{2}V_a^2 + \tfrac{1}{4}((V_a^{ij})^2 + (A_a^{ij})^2 - (M^{ij})^2 - (N^{ij})^2)$$
$$+ \bar{\lambda}^i(2\chi^i + 2\not{D}^P\lambda^i + i\kappa\gamma_5\not{A}\lambda^i - \kappa\sigma \cdot T^{ij}\lambda^j)\}. \tag{4.8}$$

According to their field equations certain auxiliary fields in (4.8) will be proportional to $(1-2\kappa^2|A|^2)^{-1}$, and upon elimination they will induce non-polynomial modifications of the type found in [14].

We now discuss the elimination of some of the auxiliary fields and the resulting non-polynomial terms in the action. The axial vector field A_a couples minimally to the scalars A, so that the equation of motion for A_a gives

$$A_a = \frac{\kappa}{1-2\kappa^2|A|^2} A\overleftrightarrow{\partial}_a A^* + \text{spinor terms}\,. \tag{4.9}$$

The vector field V_a couples to $|A|^2$ through the $D^P \cdot V$ term in (4.7). Therefore we find using the appropriate terms of (4.8)

$$V_a = \frac{2\kappa}{1-2\kappa^2|A|^2}\,\partial_a|A|^2 + \text{spinor terms}\,. \tag{4.10}$$

When inserted back into the action, (4.9) and (4.10) lead to non-polynomial modifications of the kinetic term of the scalar fields. Varying the combined action with respect to χ^i leads to a solution for the auxiliary spinor λ^i:

$$\lambda^i = -\frac{\kappa}{1-2\kappa^2|A|^2}(A\Psi^i + A^*\Psi_i)\,. \tag{4.11}$$

Substituting this into the action leads to a modification of the kinetic term of Ψ^i.

The non-polynomial modifications of the terms quadratic in the field strengths are the result of the elimination of the auxiliary tensor field t_{ab}^{ij}. This field also occurs in the Poincaré expression for T_{ab}^{ij}. The field equation for t_{ab}^{ij} is

$$t_{abij}^{+} = \frac{4}{1-2\kappa^2A^2}(\kappa A F_{ab}^{+}(V) - \kappa^2 A^2 F_{abij}^{+}(B))\,, \tag{4.12}$$

where $F_{ab}^{+}(V)$ and $F_{abij}^{+}(B)$ are the self-dual components of the field-strength tensors of V_μ and B_μ^{ij}, the gauge fields of the vector multiplet and the Poincaré theory, respectively.

A complete comparison with the results of [14] requires several field redefinitions, but the general characteristics of eqs. (4.9)–(4.12), in particular the presence of the factors $(1-2\kappa^2|A|^2)^{-1}$ and $(1-2\kappa A^2)^{-1}$, indicate that our results are indeed equivalent. To couple several vector multiplets to supergravity is of course straightforward in our approach. As compared to $N=1$, there are two new elements that lead to the emergence of non-polynomialities. First the tensor field, which is essential to obtain the modified kinetic terms of the vector fields; secondly the quantity D_C which is responsible for most of the non-polynomial terms in the solutions of the various auxiliary fields.

5. Conclusions

In this paper we have presented the superconformal tensor calculus for chiral superfields. Apart from the action of the vector multiplet, which we gave as an example, there exists a large variety of invariant actions on the basis of $N=2$ chiral superfields. Not all of them can be treated strictly within the superconformal context. However, this poses no essential limitation, since our results can be rewritten in terms of Poincaré components. This then defines a corresponding tensor calculus, which allows the construction of actions with arbitrary Weyl weight [5]. In this way one can construct a self-interaction of the vector multiplet, which is known to exist from a reduction of the SO(8) extended theory. Also the SO(2) supergravity action itself can be obtained from the unit chiral multiplet, i.e., the multiplet with only a constant A-component different from zero. This construction of the Poincaré theory, which was already mentioned in [6], is suggested by the density formula (3.3). For the unit multiplet this formula contains a term $(T_{abij}^{+})^2$ that partly coincides with the tensor terms in the Poincaré action, so that the full supersymmetric density written in Poincaré notions should lead to this action.

The $N=2$ superconformal Weyl theory can be constructed on the basis of a reduced chiral multiplet as well, but this requires a slight extension of our results. The reason is that the superconformal chiral field is no longer a Lorentz scalar but a self-dual antisymmetric Lorentz tensor. The low-θ components of this superfield are completely known. They consist of the conformal fields T^{+}_{abij}, χ^{i}_{C}, D_{C}, and the appropriately modified superconformal curvatures. These curvatures and their transformation properties can be found in [1]. Another action based on chiral multiplets was presented long ago in [15]. In this case the superfield is not reduced and one needs to determine a kinetic multiplet in order to generalize the results of [15] to local supersymmetry.

An important conclusion of our work is that the use of the superconformal algebra is extremely helpful as an intermediate step, even although this algebra is more complicated in our case than for $N=1$ [16]. It may be that this intermediate step is not possible for all multiplets. One then has to aim directly for a representation of the Poincaré algebra, which is substantially more complicated. However, the general strategy is always the same. Namely, one extends the results of rigid supersymmetry to a full representation of the super-Poincaré or super-Weyl algebra. To find the multiplication rules and the corresponding invariant densities is then straightforward.

For two of us (B.dW. and J.W.v.H.) this work is part of the research program of the Stichting voor Fundamenteel Onderzoek der Materie (F.O.M.).

References

[1] B. de Wit, J.W. van Holten and A. Van Proeyen, Nucl. Phys. B167 (1980) 186 (E: 172 (1980) 543)
[2] E.S. Fradkin and M.A. Vasiliev, Nuovo Cim. Lett. 25 (1979) 79; Phys. Lett. 85B (1979) 47
[3] B. de Wit and J.W van Holten, Nucl. Phys. B155 (1979) 530
[4] S. Ferrara and P. van Nieuwenhuizen, Phys Lett. 76B (1978) 404
[5] B. de Wit, J.W. van Holten and A. Van Proeyen, Leiden preprint, in preparation
[6] P. Breitenlohner and M. Sohnius, Nucl. Phys. B165 (1980) 483;
P. Breitenlohner, *in* Supergravity, Proc. Supergravity workshop, ed. P. van Nieuwenhuizen and D.Z. Freedman (North-Holland, Amsterdam, 1979)
[7] P. Fayet, Nucl. Phys. B113 (1976) 135
[8] R. Grimm, M. Sohnius and J. Wess, Nucl. Phys. B133 (1978) 275
[9] A. Salam and J. Strathdee, Nucl. Phys. B76 (1979) 477; Phys. Rev. D11 (1975) 1521;
S. Ferrara, J. Wess and B. Zumino, Phys. Lett. 51B (1974) 239
[10] B. de Wit, Nucl. Phys. B158 (1979) 189
[11] P. H. Dondi and M. Sohnius, Nucl. Phys. B81 (1974) 317
[12] R.J. Firth and J D. Jenkins, Nucl. Phys. B85 (1975) 525
[13] B. de Wit and D.Z. Freedman, Phys. Rev. D12 (1975) 2286,
P. Breitenlohner, Phys. Lett 67B (1977) 49
[14] J.F. Luciani, Nucl. Phys. B132 (1978) 325
[15] D.M. Capper and G. Leibbrandt, Nucl. Phys. B85 (1975) 503
[16] M. Kaku, P.K. Townsend and P. van Nieuwenhuizen, Phys. Rev. D17 (1978) 3179